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A New Approach to Build a Heat Flux Model of Milling Processes

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Abstract

During the finishing milling process of thin-walled workpieces, deformations occur due to the mechanical and thermal influences. Hence, the manufacturing quality may be affected. The aim of a research project at the *iwb* is to simulate thermally caused deformations and deriving methods to reduce them. Two approaches subsume the term "reduction methods": the use of less deformation-causing process parameters or alternatively – if this approach is insufficient – to compensate the deformation.
Simulating thermally induced deformations necessitates a description of the heat input, which is caused by the milling process. Furthermore, this description needs to be valid for the whole spectrum of process parameters recommended for the used tool. The model of the heat source is based on milling experiments with an in-process measurement of the thermally caused deformations. The experimental results allow differentiating between thermally and mechanically induced deformations due to the restriction on an end milling finishing process. However, these measurements only provide a relation between process parameters (variation of cutting speed v_c, feed per tooth f_z and width of cut a_e) and deformations. The simulation model, which will be described within this paper in detail, provides the relation between a variable heat input and its resulting deformations. The experimental and the simulative relations together are leading to a heat source model in dependence of the process parameters.

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1. Introduction

During the milling process of complex thin-walled workpiece structures, deviations in shape and dimension [1] occur due to its mechanical and thermal influences. These may affect the quality of the machining result negatively or even violate tolerance specifications. In particular, engine [2] and structural [3] components in aerospace industry (with removal rates up to 95 %) as well as engine heads and gearboxes [4] in automotive industry represent such thin-walled workpiece structures. Economical and ecological aspects lead to an increased use of dry machining operations within these industry sectors [5]. Thus, the influence of thermal induced deformations as well as its modeling becomes more important [6]. The determination of process heat flux models still presents a major challenge in manufacturing science [7, 8].

Within this research project, the global objective is to describe the mechanically and thermally induced deformations of any machinable workpiece structure by simulation (limited to a milling process with a helical end mill cutter). In order to get short computational time ARRAZOLA ET AL. [7] states hybrid models as the required modeling strategy. Here, a numerical model of the workpiece structure and an empirical process model are used. Based on this simulation, compensation strategies will be derived, which should at best not affect the material removal rate.

This publication mainly comprises a description of the process-heat-source model as one part of the whole approach as well as a short description of the structure and the proceeding of the simulation model.

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Nomenclature

$\boldsymbol{a}$	area of active surfaces associated to the heat fluxes in vector $\boldsymbol{\lambda}_\Delta$
a_e	cutting width
a_p	cutting depth
c	model constant
b_W	remaining cantilever thickness
d	tool diameter
f_z	feed rate per tooth z
h_{sp}	cantilever height
i, j	counter of independent parameters
J_T	cost function of thermal regression models
k	model number of the regression model
F_y	active component of process forces (in y-direction)
m_k	row size of vectors and matrices
n_k	column size of matrices
n	rotational speed of the tool
p	position dependency of the regression model
$\bar{p}$	number of discrete position points
P_c	cutting performance
Q_w	material removal rate
$\dot{q}$	heat flux
t	time dependency of the regression model
$\bar{t}$	number of discrete time points
u_F	mechanically caused deformation of the workpiece
u_T	thermally caused deformation of the workpiece
v_c	cutting speed
v_f	feed speed
x	studentized independent parameter
z	number of tool teeth
z_{MP}	position of the measuring point (MP)
Δ	difference of two models (k)
$\boldsymbol{\lambda}$	vector of studentized independent parameters
$\boldsymbol{\varphi}$	linear model constants
$\boldsymbol{\Omega}$	quadratic and linear interacting model constants

2. Characterization of the milling process

Deformations of the workpiece are mechanically (process forces) and thermally (heat input from shearing, friction and separation within the cutting zone) caused. Depending on the static stiffness of the thin-walled structure, the process forces induce one part of the deformation (Fig. 1, left). The second part results from a heat induced inhomogeneous temperature field, which in turn affects thermal expansions over the workpieces cross-section (Fig. 1, right).

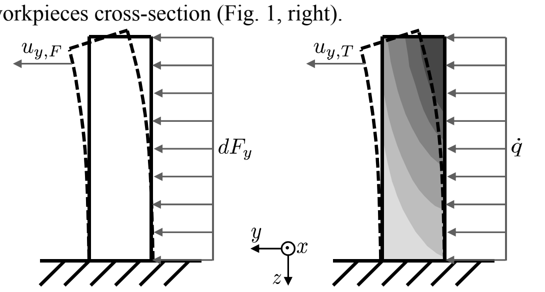

Fig. 1.: Schematic representation of mechanically (left part) and thermally (right part) caused deformations, using the example of a cantilever beam.

In the literature, the process forces and the resulting deformations of the workpiece and also of the tool are described quite extensively (e. g. [3, 9-12]). Therefore, the forces will not be addressed in detail within this publication. This research project uses the process force model developed by ALTINTAS [13] to simulate mechanically caused deformations. Research about the effects of the process heat is rarer. Therefore, the DFG (German Research Foundation) funded a priority program (abbr. SPP) called "Modeling, Simulation and Compensation of Thermal Effects for Complex Machining Processes". Several machining operations with focus on the resulting deviation in terms of shape and dimension shall be examined within the SPP. The focus of this project, which is also part of the SPP, is milling of thin-walled complex workpieces with a helical end mill cutter.

Unlike the process forces, the heat flux into the workpiece is not measurable directly [8]. Furthermore, the milling tool prevents the accessibility of the point of interest. Thus, the heat flux needs to be generally determined indirectly from its resulting effects: the workpieces temperature (which can be measured at one or more discrete points) or alternatively its deformation behavior. Such tasks are called inverse heat transfer problems. JOLIET ET AL. [14], DENKENA ET AL. [15] and SÖLTER & GULPAK [5] have solved the inverse heat transfer problem (IHTP) by comparing simulated and measured temperatures for various milling processes. OZISIK [16] notes the general ill conditioning of IHTP. The ill conditioning of IHTP (in sense of HADAMARD [17]) "means that small disturbances of the boundary data (the temperature [...]) cause large solution errors and oscillations in the heat flux density" [18]. To get a better conditioning of the inverse problem, in this publication the deformation is used to determine the heat input. This approach promises high model accuracy, because the whole transient spatial temperature field inside the workpiece causes its time-varying deformation and will be considered on this way, not only the surface temperatures.

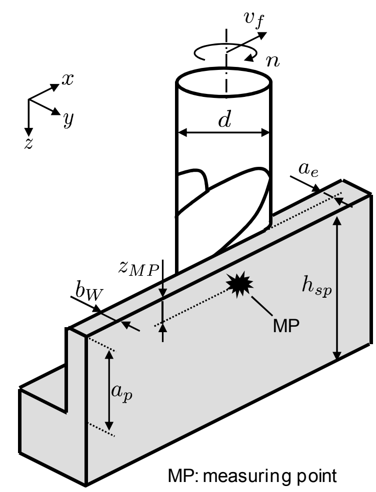

Fig. 2.: Schematic representation of the process being discussed within this publication.

Finally, higher conditioned process heat models can be expected. The method described in LOEHE ET AL. [19] and LÖHE & ZÄH [20] to separate measured deformations into their mechanical and thermal part form the basis to solve the IHTP. This is valid for finishing milling processes (like shown in Fig. 2), which indeed are causative for the final shape of the workpiece. Fig 2 visualizes the set-up of the examined process.

3. Global approach

As preliminarily mentioned, the deformations should be reduced by means of simulation methods. Therefore, an analytical model of the process is coupled with a numerical model of the workpiece structure. The analytical model describes the forces and the heat source in dependence of the process parameters and the contact conditions while the numerical model (in the present case a Finite-Element-(FE)-Model) is used to calculate the reaction of the workpiece structure. There are several advantages of using a numerical simulation for such approaches: any machinable workpiece geometry can be calculated (important in terms of heat conduction, thermal and mechanical boundary conditions) and temperature-dependent material parameters as well as plasticity can be applied. The analytical process model is coupled with the FE-model as a boundary condition using MSC.Marc as FE-software. With this simulation method specific scenarios to reduce the occurring deformations can be formulated. There are two general approaches that can be distinguished: process-related and compensation strategies. Due to its simplicity, the process-related approaches (Fig. 3, primary action) are to be preferred over the compensation techniques (Fig. 3, secondary action). Fig. 3 shows the general procedure to reduce process caused deformations.

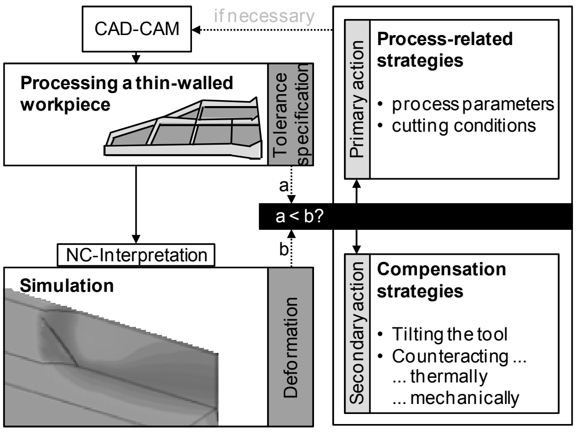

Fig. 3.: Procedure model to reduce process caused deformation.

3.1. Process-related approach to reduce deformations

The process-related approaches aim to identify one or more combinations of process parameters and contact conditions that are resulting in a minimum of the deformation. Changed process parameters can be integrated into the NC-Code directly, whereas adapted contact conditions may need a replanning of the tool-path (e.g. with a CAD-CAM-Software). The following process parameters can be changed:

- the cutting speed v_c and
- the feed rate per tooth f_z.

Changing the cutting conditions is characterized by a mutual adaption of

- the cutting width a_e and
- the cutting depth a_p.

3.2. Compensation strategies to reduce deformations

There are many different possibilities to compensate occurring deformations like tilting the cutting tool or additional actuators compensating the deformation with specific counter forces or counter heat sources. Unlike the process-related approaches, the implementation of compensation strategies requires a high technical effort and in the majority of cases it requires individual solutions.

However, the performance of the analytical process description has a decisive influence on the quality of the deformation simulation. Therefore, the following chapter describes the analytical heat source model in detail.

4. Heat source model

While milling complex workpieces, various combinations of process parameters may occur along the tool path (e. g. generated by CAD-CAM). Furthermore, the previously mentioned compensation algorithm may cause additional combinations of process parameters. For this reason, the process model has to be valid within a wide range of parameters. To fulfill this requirement, the whole field of parameters, recommended by the tool manufacturers for finishing processes, was used to calibrate the heat source model.

As discussed in chapter 2, especially the heat input into the workpiece is focused in the following section. The workpiece's deflection during a stationary milling process (Fig. 2), especially its thermal part, represents the basis for the determination. The model of the heat source $\dot{q}$ can be formulated in general with formula (1):

$$\dot{q} = f(a_e, f_z, v_c, \dots) \qquad (1)$$

There is no thermally caused deformation u_T within this formulation and determining the heat source from deformation measurements directly seems to be difficult. Nevertheless, using design of experiments and applying a regression analysis leads to formula (2), describing the measured thermally caused deformation $u_{T,Measure}$ in dependence of the process parameters:

$$u_{T,Measure} = f(a_e, f_z, v_c, \dots) \qquad (2)$$

To get the missing step between the measured deformation and the heat input, simulations are used. Therefore, each parameter setup of the measurements was built up virtually and was loaded with different dimensioned heat fluxes.

Analogously to the measurement data, a regression analysis can be applied to the simulation results:

$$u_{T,Simulate} = f(a_e, f_z, v_c, \dot{q}, \ldots). \qquad (3)$$

A comparison of formula (2) and (3) leads to:

$$\begin{aligned} u_{T,Measure} &\stackrel{!}{=} u_{T,Simulate} \\ 0 &\stackrel{!}{=} u_{T,Simulate} - u_{T,Measure} = u_{T,\Delta} \end{aligned} \qquad (4)$$

Depending on the model's complexity, there is one or more $\dot{q}$ satisfying formula (4). Basically, the solution represents the general form (formula (1)).

WARNECKE [21] already describes multiple interaction zones in his elaboration on the topic of chip formation. Within those zones shearing, friction and separation processes generate heat. Depending on the position of each heat source and on the process parameters, different amounts of heat are reaching the workpiece structure (depending on the distribution of heat flow, described by DAWSON & MALKIN [22]). All zones that may influence the workpiece need to be considered while satisfying formula (4). Here, mainly the primary and the secondary zone are crucial [23].

The following section describes the procedure from formulas (1) to (4) in detail, considering multiple heat sources. The nomenclature for the regression analysis is based on SIEBERTZ ET AL. [24]. Here, regression models of the measured ($k = 1$) and simulated ($k = 2$) thermally caused deformation ($u_{T,k}$) are generally described for one point of comparison (time (t) and one measuring position (p, e. g. MP in Fig. 2)) as follows:

$$\begin{aligned} u_{T,k}(t,p) = {} & c_{0,k}(t,p) + \\ & + c_{1,k}(t,p)x_1 + \cdots + c_{i,k}(t,p)x_i + \\ & + c_{1,1,k}(t,p)x_1x_1 + \cdots + c_{i,j,k}(t,p)x_ix_j, \end{aligned} \qquad (5)$$

with $i \leqq j$.

According to this, for each characteristic point in time and each measuring position that should be considered, an additional model needs to be built. The regression model shown in formula (5) implies linear and quadratic parts as well as linear interactions between the independent parameters (x_i, resp. x_j). As pre-tests have shown, this model order will be sufficient to model the behavior of a milling process. Further steps are requiring a matrix format. The following vectors resp. matrices are describing the system:

$$\boldsymbol{\lambda}_k = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_i \end{bmatrix}; \quad \boldsymbol{\varphi}_k(t,p) = \begin{bmatrix} c_{1,k}(t,p) \\ c_{2,k}(t,p) \\ \vdots \\ c_{i,k}(t,p) \end{bmatrix};$$
$$\boldsymbol{\Omega}_k(t,p) = \begin{bmatrix} c_{1,1,k}(t,p) & c_{1,2,k}(t,p) & \cdots & c_{1,j,k}(t,p) \\ 0 & c_{2,2,k}(t,p) & \cdots & c_{2,j,k}(t,p) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & c_{i,j,k}(t,p) \end{bmatrix}. \qquad (6)$$

Now, formula (5) can be written in matrix format, as formula (7) shows:

$$u_{T,k}(t,p) = \boldsymbol{\lambda}_k^T \boldsymbol{\Omega}_k(t,p) \boldsymbol{\lambda}_k + \boldsymbol{\lambda}_k^T \boldsymbol{\varphi}_k(t,p) + c_{0,k}(t,p) \qquad (7)$$

Here, the size of vector $\boldsymbol{\lambda}_1$ ($m_1 \times 1$) equals the number of independent parameters considered in the measurement model (see formula (2) and formula (5) for $k = 1$), whereas the size of $\boldsymbol{\lambda}_2$ ($m_2 \times 1$) is increasing with the number of varying parameters $\boldsymbol{\lambda}_\Delta$ (s. formula (8), e. g. multiple heat sources $\dot{q}$ in the simulation model):

$$\boldsymbol{\lambda}_2 = \underset{m_2 \times 1}{\begin{bmatrix} \boldsymbol{\lambda}_1 \\ \boldsymbol{\lambda}_\Delta \end{bmatrix}} \qquad (8)$$

The same extension is applied for the size of vector $\boldsymbol{\varphi}_k$ and also for the square matrix $\boldsymbol{\Omega}_k$. Therefore, the size of $\boldsymbol{\varphi}_1$ and $\boldsymbol{\Omega}_1$ need to be expanded by zeros to the size of $\boldsymbol{\varphi}_2$ and $\boldsymbol{\Omega}_2$, which results in $\widetilde{\boldsymbol{\varphi}}_1$ and $\widetilde{\boldsymbol{\Omega}}_1$ (formula (9)). Then, $\boldsymbol{\lambda}_1$ can be set equal to $\boldsymbol{\lambda}_2$ without any further restrictions.

$$\widetilde{\boldsymbol{\varphi}}_1 = \underset{m_2 \times 1}{\begin{bmatrix} \boldsymbol{\varphi}_1 \\ \mathbf{0} \end{bmatrix}}; \quad \widetilde{\boldsymbol{\Omega}}_1 = \underset{m_2 \times n_2}{\begin{bmatrix} \boldsymbol{\Omega}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}} \qquad (9)$$

Subsequently, formula (7) can be expressed as shown in formula (10) for $k = 1$.

$$u_{T,1}(t,p) = \boldsymbol{\lambda}_2^T \widetilde{\boldsymbol{\Omega}}_1(t,p) \boldsymbol{\lambda}_2 + \boldsymbol{\lambda}_2^T \widetilde{\boldsymbol{\varphi}}_1(t,p) + c_{0,1}(t,p) \qquad (10)$$

Using the condition from formula (4) and symbolizing the difference of both models $k = 1$ and $k = 2$ with subscript Δ, the following formulation occurs:

$$\begin{aligned} 0 &\stackrel{!}{=} \boldsymbol{\lambda}_2^T \boldsymbol{\Omega}_\Delta(t,p) \boldsymbol{\lambda}_2 + \boldsymbol{\lambda}_2^T \boldsymbol{\varphi}_\Delta(t,p) + c_{0,\Delta}(t,p) \\ &= u_{T,\Delta}(t,p,\boldsymbol{\lambda}_2) \end{aligned} \qquad (11)$$

Analogously to formulas (2) to (4) one or more $\boldsymbol{\lambda}_2$ are satisfying formula (11) for all discrete points in time (t) and all positions (p). A cost function summarizes this requirement:

$$J_T(\boldsymbol{\lambda}_2) = \sum_{t=1}^{\bar{t}} \sum_{p=1}^{\bar{p}} \left(u_{T,\Delta}(t,p) \right)^2 \qquad (12)$$

Searching a $\boldsymbol{\lambda}_2$ minimizing formula (12) constitutes a classical optimization problem (13). A squared sum is used here in order to satisfy the optimization formula (11) to be zero:

$$\min_{\boldsymbol{\lambda}_2}(J_T) \qquad (13)$$

To limit the amount of possible solutions, boundary conditions need to be added to the optimization problem. The first boundary condition considers the vector $\boldsymbol{\lambda}_1$ as a known quantity of vector $\boldsymbol{\lambda}_2$. Thus, only $\boldsymbol{\lambda}_\Delta$ will be changed in the optimization.

$$\boldsymbol{RB1}: \quad \boldsymbol{\lambda}_1 = [a_e \quad f_z \quad v_c \quad ...]^T \tag{14}$$

Within the second boundary condition, only heat sources and no heat sinks are allowed to be a solution of formula (13):

$$\boldsymbol{RB2}: \quad \boldsymbol{\lambda}_\Delta \geq \mathbf{0} \text{ (elementwise)} \tag{15}$$

$\mathbf{0}$ represents a zero vector with the same size as $\boldsymbol{\lambda}_\Delta$. The last boundary condition ensures the solution to be plausible by calculating a power balance of the whole process:

$$\boldsymbol{RB3}: \quad \boldsymbol{\lambda}_\Delta^T \boldsymbol{a} \leq P_c \tag{16}$$

With vector $\boldsymbol{a}$ representing the area of the active surfaces of each heat flux from $\boldsymbol{\lambda}_\Delta$, the summarized products of heat flux and the active surface area must never exceed the cutting performance P_c of the process. Because only the heat flux into the workpiece is considered here, the cutting performance may never be reached completely. To calculate the cutting performance, [25] or [26] can be used:

$$P_c = k_c \cdot Q_w = F_c \cdot v_c. \tag{17}$$

Therein, the specific cutting energy k_c, resp. the cutting force F_c are calculated analogously to DENKENA & TÖNSHOFF [25], KÖNIG ET AL. [27] or ALTINTAS [13].

While using all boundary conditions, the optimization (formula (13)) will result in values of $\boldsymbol{\lambda}_\Delta$, which are plausible for the examined tool-workpiece combination.

5. Simulation model

The simulation model has a dual purpose: on the on hand, to get the connection between measured deformation and the heat flux (chapter 4), on the other hand, to represent the numerical part of the hybrid process model (mentioned in chapter 1), used to simulate the local and global deformation behavior of the workpiece.

The basic strategy of meshing is visualized in Fig. 4. The segment of the hollow cylinder (Fig. 4, no. 1) partitions the geometry into process-near (Fig. 4, no. 2) and process-distant (Fig. 4, no. 3) regions. Because of high temperature gradients, large deformations and detailed geometric characteristics that are expected within the process-near region, it is meshed much finer and is further divided by extrapolations of the current (Fig. 4, no. 6 (t)) and the last (Fig. 4, no. 6 (t-1)) cutting faces in order to load the numerical workpiece structure with a geometrical defined heat flux (Fig. 4, no. 4). This heat flux represents the effect of ploughing during one time step. A second heat flux acts on the edge of the workpiece (Fig. 4, no. 5) built by the cutting plane, representing shearing, friction and separation within the shearing zone [21]. Due to the process, removing the volume of the workpiece successively (chip removal), a huge amount of this heat is removed with the chip.

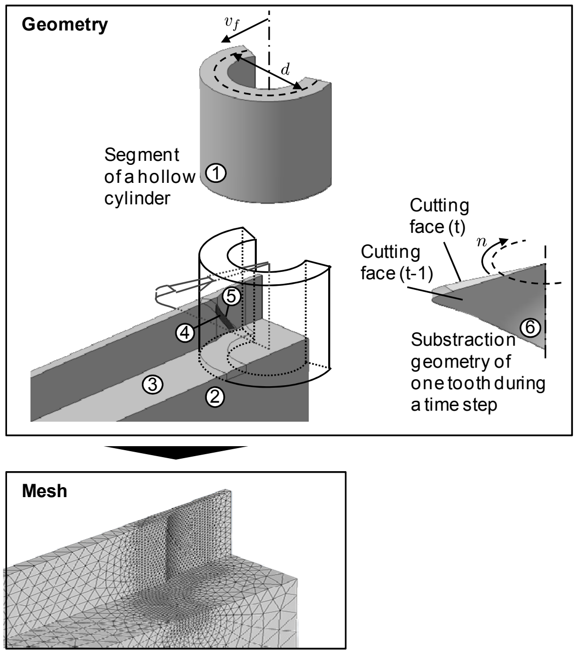

Fig. 4.: Schematic representation of partitioning the geometry for meshing.

Figure 5 visualizes the specific structure and procedure of the simulation model, as implemented in this research.

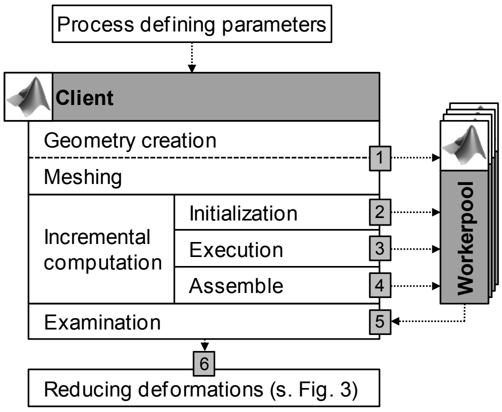

Fig. 5.: Structure of the client's procedure (image source: MathWorks)

However, many alternative approaches using different software tools are also possible. A so-called client (Fig. 5) is used to manage all data, to assign jobs to a pool of workers every increment as well as to examine a successful execution of the assigned jobs (no. 5). Here, MATLAB is used as client and also as worker software. Single processors of one or more computers are used as workers. Based on the model's input, jobs are created by the client. As shown in Figure 5, several different jobs are to be distinguished for each increment:

- create and mesh the workpiece geometry (no. 1)
- initialize the computation job (mapping pre-states) (no. 2)
- execute the computation (no. 3)
- assemble the incremental results to one result file (no. 4)

Approaches to reduce the deformations (no. 6) are part of the client, because they may affect changes of the simulation procedure (e. g. repeating increments). The presented approach enables a parallelization of modules 1 to 4 (Fig. 5) and thus leads to improved performance.

With regard to the heat flux regression model (chapter 4), the simulation of multiple combinations of process parameters and heat sources can further be parallelized well.

6. Conclusion and Perspectives

Within this publication, an approach to derive a process heat flux from thermally caused deformations was demonstrated. It was a central aim of this work to improve the conditioning of this IHTP by using deformation measurements. Considering temperatures and deformations together will additionally increase the conditioning of the IHTP. Furthermore, the structure and procedure of the simulation model was demonstrated.

To improve the applicability of the whole approach, the complexity of a full factorial experimental design has to be reduced. Two steps are applied to do this:

- Further design of experiments should consider the influence of known independent parameters $\boldsymbol{\lambda}_1$ (linear, quadratic or linearly interacting).
- Some of the model constants $c_{i,\Delta}$, resp. $c_{i,j,\Delta}$ may be assignable to physical parameters. In consequence, calibrating them is not necessary any longer.

Both measures were reducing the amount of model constants and thus the amount of measurements for calibration. Besides the conditioning of the IHTP, the heat flux model as part of the hybrid process model has the potential to predict thermally caused deformation in a wide range of process parameters with high accuracy. This may lead to fewer rejections within the initially named (chapter 1) industry and further contributes to the idea of quality management: “right first time”.

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